Instability and dynamics of two nonlinearly coupled laser beams in a two-temperature electron plasma

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We consider nonlinear interactions between two colliding laser beams in an electron plasma, accounting for the relativistic electron mass increase in the laser fields and radiation pressure driven electron-acoustic (EA) perturbations that are supported by hot and cold electrons. By using the hydrodynamic and Maxwell equations, we obtain the relevant equations for nonlinearly coupled laser beams and EA perturbations. The coupled equations are then Fourier analyzed to obtain a nonlinear dispersion relation. The latter is numerically solved to show the existence of new classes of the parametric instabilities in the presence of two colliding laser beams in a two-electron plasma. The dynamics of nonlinearly coupled laser beams in our electron plasma is also investigated. The results should be useful in understanding the nonlinear propagation characteristics of multiple electromagnetic beams in laser-produced plasmas as well as in space plasmas.

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I. INTRODUCTION

There has been a growing interest in studying nonlinear interactions between intense electromagnetic beams and plasmas that are composed of hot and cold electrons. Such two-electron plasmas are encountered both in laboratories [1,2] and in space [3]. In such plasmas, we have the possibility of electron-acoustic waves [4] which can nonlinearly interact with high-frequency waves. Some Cluster observations of high-frequency waves in the terrestrial foreshock have been interpreted as the nonlinear decay of Langmuir waves into ion-acoustic and electron-acoustic waves [5]. On the other hand, interactions between a coherent single electromagnetic pulse and electron-acoustic waves can produce interesting nonlinear effects [6–8] in a two-electron plasma. In laboratory plasmas, the hot electron component is produced by collisionless heating involving the Raman backscattering instability [9]. Theoretical and simulation studies have shown that we can have nonlinear interactions between the laser light and kinetic structures and/or electron holes [10–12] and beam-acoustic modes [13]. Furthermore, if the amplitude of the laser light is large enough, we also have new effects due to the relativistic electron mass increase in the electromagnetic fields [14–20,24]. The importance of interplay between the relativistic electron mass variation nonlinearity and the radiation pressure driven density fluctuations has been recognized [14,15,21] in the context of numerous parametric instabilities of a single laser beam in plasmas.

The presence of multiple laser beams in a plasma can give rise to a new set of interesting phenomena [22–25]. One of the potential applications of two colliding laser beams in plasmas is the excitation of large amplitude Langmuir waves, which in turn accelerate electrons to ultrarelativistic speeds [23]. The coupling between two electromagnetic waves in plasmas can be described by a system of coupled nonlinear Schrödinger equations that describe nonlinear interactions between localized light [25,26] and Langmuir or ion-acoustic waves. Laser beams that are copropagating at a small distance from each other sometimes spiral around each other [27] or merge [28]. At relativistic intensities, laser beams can give rise to fast plasma waves via higher-order nonlinearities [22,23,29], or via the beat wave excitation at frequencies different from the electron plasma frequency [30]. Particle-in-cell simulations [31] show that large-amplitude electron plasma waves can be excited by colliding laser pulses, or by two copropagating electromagnetic pulses where a long trailing pulse is modulated efficiently by the periodic plasma wake behind the heading short laser pulse [32]. The effects on parametric instabilities of a partially incoherent pump wave (with a distribution of wave modes) was investigated both theoretically [33] and experimentally [34], where it was found that the effect of finite bandwidth is, in general, to increase the instability thresholds and lower the growth rate. The reason is that the pump energy is distributed over the bandwidth of the pump, while only the energy within the resonance width contributes to the instability.

In this paper we consider nonlinear interactions between two colliding electromagnetic beams and a plasma that is composed of immobile ions and inertialess hot electrons and cold inertial electrons. We use the Maxwell and Poisson equations, in conjunction with the hydrodynamic equations for the electrons, to derive a set of three coupled equations that show nonlinear couplings between two laser beams and electron-acoustic (EA) perturbations that are supported by two distinct groups of electrons in our plasma. In our investigation, we account for the combined action of the relativistic electron mass increase in the electromagnetic fields, as well as the electron density perturbation associated with EA perturbations that are reinforced by the radiation pressure of two laser beams. The coupled equations are then Fourier analyzed to obtain a nonlinear dispersion relation that is appropriate for investigating the parametric instabilities. The nonlinear dispersion relation is numerically analyzed to demonstrate the existence of new classes of parametric interactions caused by two electromagnetic pump waves. We also numerically solve the coupled nonlinear equations to investigate the features of spatiotemporal evolution of nonlinearly interacting laser beams and EA perturbations.

II. NONLINEAR MODEL EQUATIONS

Let us derive the set of equations that govern the dynamics of two nonlinearly coupled laser beams and driven EA perturbations in a two-electron plasma. At equilibrium, we have $n_{i0}=n_0=n_{0c}+n_{0h}$, where n_{i0} is the equilibrium ion number density, and n_{0c} and n_{0h} are the number densities of the cold and hot electron components, respectively. We will assume that ions are immobile and homogeneously distributed. In the presence of laser beams, the density and velocity perturbations of the cold electrons, supporting the EA perturbations, are governed by the continuity equation

$$\frac{\partial n_{sc}}{\partial t}+n_{0c}\boldsymbol{\nabla}\cdot\mathbf{v}_{sc}=0, \tag{1}$$

and the momentum equation

$$m\frac{\partial \mathbf{v}_{sc}}{\partial t}=e\nabla\phi_s-\frac{m}{2}\nabla\langle\mathbf{v}_f^2\rangle, \tag{2}$$

where $n_{sc}(\ll n_{0c})$ and v_{sc} are the number density and velocity perturbations, respectively, for the cold electrons, ϕ_s is the electrostatic potential, $m\nabla\langle\mathbf{v}_f^2\rangle$ denotes the radiation pressure, $\mathbf{v}_f$ is the electron quiver velocity in the laser fields, and the angular brackets denote averaging over one laser light period. The radiation pressure term comes from the averaging of the advection and nonlinear Lorentz force terms over one light period (see Ref. [15]). Specifically, in an unmagnetized plasma we used $m\langle\mathbf{v}_f\cdot\boldsymbol{\nabla}\mathbf{v_f}\rangle+(e/c)\langle\mathbf{v}_f\times\mathbf{B}\rangle$ with $\mathbf{v}_f=e\mathbf{A}/mc$ and $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$ to obtain the second term in the right-hand side of Eq. (2). Here, m and e are the electron mass and the magnitude of the electron charge, respectively.

Since the phase velocity of the EA perturbations is much smaller than the thermal speed of the hot electron component, we obtain from the equation of motion

$$0=e\nabla\phi_s-T_h\frac{\nabla n_{sh}}{n_{0h}}-\frac{m}{2}\nabla\langle\mathbf{v}_f^2\rangle, \tag{3}$$

which gives

$$n_{sh}=n_{0h}\left(\frac{e\phi_s}{T_h}-\frac{m\langle\mathbf{v}_f^2\rangle}{2T_h}\right), \tag{4}$$

where n_{sh} ($\ll n_{0h}$) is the small density perturbation and T_h the equilibrium temperature of the hot electrons. We have here assumed that the background hot electrons are Boltzmann distributed and that the electron distribution is not modified significantly by the electrostatic waves. In the opposite case, a kinetic treatment is needed [13].

The electrostatic potential is obtained from the Poisson equation

$$\nabla^2\phi_s=4\pi e(n_{sc}+n_{sh}). \tag{5}$$

Combining Eqs. (1), (2), (4), and (5), we have

$$\left[(\nabla^2-k_{Dh}^2)\frac{\partial^2}{\partial t^2}+\omega_{pc}^2\nabla^2\right]\phi_s=\frac{m}{2e}\left(\omega_{pc}^2\nabla^2-k_{Dh}^2\frac{\partial^2}{\partial t^2}\right)\langle\mathbf{v}_f^2\rangle, \tag{6}$$

where we have denoted $\omega_{pc}=(4\pi e^2 n_{c0}/m)^{1/2}$ and $k_{Dh}=(4\pi n_{h0}e^2/T_h)^{1/2}$. Equation (6) is the driven (by the radiation pressure) EA perturbations. We see that the nonlinear coupling between the latter and the laser beams occurs only when the space charge electric field associated with the EA perturbations is reinforced by the laser beam pressure.

We next derive the governing equation for intense laser beams in a two-component electron plasma that supports the EA perturbations. The circularly polarized laser beam propagation is governed by the Maxwell equation

$$\boldsymbol{\nabla}\times\mathbf{B}=-\frac{4\pi}{c}(n_0+n_{sc}+n_{sh})e\mathbf{v}_f+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}. \tag{7}$$

We now express the electromagnetic fields in terms of the vector potential $\mathbf{A}$ as $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$ and $\mathbf{E}=-(1/c)\partial\mathbf{A}/\partial t$. Since $\partial\mathbf{p}_e/\partial t=-e\mathbf{E}$, we have for the relativistic electron momentum $\mathbf{p}_e=e\mathbf{A}/c$. By using the definition $\mathbf{p}_e=m\gamma\mathbf{v}_f$, where $\gamma=(1-\mathbf{v}_f^2/c^2)^{-1/2}$ is the relativistic gamma factor, we have for circular polarized laser beams

$$\mathbf{v}_f=\frac{e\mathbf{A}}{mc}\left(1+\frac{e^2|\mathbf{A}|^2}{m^2c^4}\right)^{-1/2}. \tag{8}$$

For weakly relativistic electrons, i.e., $e^2|\mathbf{A}|^2/m^2c^4\ll 1$, we can approximate Eq. (8) by

$$\mathbf{v}_f\approx\frac{e\mathbf{A}}{mc}\left(1-\frac{e^2|\mathbf{A}|^2}{2m^2c^4}\right). \tag{9}$$

With these prerequisites, and using Eq. (5) to reduce n_{sc} and n_{sh}, Eq. (7) becomes

$$\left(\frac{\partial^2}{\partial t^2}-c^2\nabla^2\right)\mathbf{A}+\frac{\omega_{\mathrm{p0}}^2}{n_0}\left(n_0+\frac{\nabla^2\phi_s}{4\pi e}\right)\mathbf{A}-\omega_{\mathrm{p0}}^2\frac{e^2|\mathbf{A}|^2}{2m^2c^4}\mathbf{A}=0. \tag{10}$$

In the presence of two laser beams, we divide the vector potential into two parts according to $\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2$ so that $|\mathbf{A}|^2=|\mathbf{A}_1|^2+|\mathbf{A}_2|^2+2\mathbf{A}_1\cdot\mathbf{A}_2$. If $\mathbf{A}_1\cdot\mathbf{A}_2\neq 0$, there will be beat waves in the plasma on the sum and difference frequencies $\Delta\omega_+=\omega_1+\omega_2$ and $\Delta\omega_-=\omega_1-\omega_2$ and the respective wave vectors $\Delta\mathbf{k}_+=\mathbf{k}_1+\mathbf{k}_2$ and $\Delta\mathbf{k}_+=\mathbf{k}_1-\mathbf{k}_2$. This is frequently used in beat-wave acceleration of electrons [23] and ionospheric heating experiments [35,36], where two electromagnetic waves are used to drive resonantly Langmuir or upper-hybrid waves whose frequencies and wave vectors equal the difference frequencies and wave vectors of the two electromagnetic waves [37]. Here, we will assume that $\mathbf{A}_1\cdot\mathbf{A}_2\ll|\mathbf{A}_1|^2+|\mathbf{A}_2|^2$ and that the beat frequency is far away from any eigenmodes in the plasma, so that the cross-coupling term $\mathbf{A}_1\cdot\mathbf{A}_2$ can be neglected. Hence, from (10) we obtain the two coupled electromagnetic wave equations

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{A}_j + \frac{\omega_{p0}^2}{n_0}\left(n_0 + \frac{\nabla^2\phi_s}{4\pi e}\right)\mathbf{A}_j - \frac{\omega_{p0}^2 e^2}{2m^2c^4}(|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2)\mathbf{A}_j = 0, \quad (11)$$

where $j=1,2$ for the two laser beams. Assuming that $\mathbf{A}_j$ is proportional to $\exp(i\mathbf{k}_j\cdot\mathbf{r} - i\omega_j t)$, where $\omega_j \gg |\partial/\partial t|$, we obtain, in the slowly varying envelope approximation, the two coupled nonlinear Schrödinger equations, namely,

$$2i\omega_j\left(\frac{\partial}{\partial t} + \mathbf{v}_{gj}\cdot\nabla\right)\mathbf{A}_j + c^2\nabla^2\mathbf{A}_j - \frac{e}{m}(\nabla^2\phi_s)\mathbf{A}_j + \frac{\omega_{p0}^2 e^2}{2m^2c^4}(|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2)\mathbf{A}_j = 0, \quad (12)$$

where we used the laser frequency $\omega_j = (\omega_{p0}^2 + c^2k_j^2)^{1/2}$, the group velocity $\mathbf{v}_{gj} = c^2\mathbf{k}_j/\omega_j$, and the plasma electron frequency $\omega_{p0} = (4\pi n_{e0}e^2/m)^{1/2}$. Introducing $\langle \mathbf{v}_f^2\rangle \approx (e^2/m^2c^2)(|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2)$ into Eq. (6), we have

$$\left[(\nabla^2 - k_{Dh}^2)\frac{\partial^2}{\partial t^2} + \omega_{pc}^2\nabla^2\right]\phi_s = \frac{e}{2mc^2}\left(\omega_{pc}^2\nabla^2 - k_{Dh}^2\frac{\partial^2}{\partial t^2}\right)(|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2). \quad (13)$$

Equations (12) and (13) are the desired set governing the nonlinear couplings between two colliding laser beams and EA perturbations in a two-electron plasma with fixed ion background.

Nonlinear dispersion relation

We now investigate the parametric instabilities of two laser beams that are nonlinearly interacting with the EA perturbations. Accordingly, we Fourier analyze (12) and (13) by assuming that $\phi_s = \hat{\phi}_s\exp(-i\Omega t + i\mathbf{K}\cdot\mathbf{r})+$ complex conjugate, while $\mathbf{A}_j = [\mathbf{A}_{j0} + \mathbf{A}_{j+}\exp(i\mathbf{K}\cdot\mathbf{r} - i\Omega t) + \mathbf{A}_{j-}\exp(-i\mathbf{K}\cdot\mathbf{r} + i\Omega t)]\exp(-i\Omega_0 t)$, where $|\mathbf{A}_{j0}| \gg |\mathbf{A}_{j\pm}|$. Sorting out for different powers of $\exp(i\mathbf{K}\cdot\mathbf{r} - i\Omega t)$, we obtain from (12) the nonlinear frequency shift

$$\Omega_{j0} = -\frac{\omega_{p0}^2 e^2}{4\omega_j m^2c^4}(|\mathbf{A}_{10}|^2 + |\mathbf{A}_{20}|^2), \quad (14)$$

and from Eqs. (6) and (12) we have a system of equations

$$D_{1+}X_{1+} + Q|A_{10}|^2(X_{1+} + X_{1-} + X_{2+} + X_{2-}) = 0, \quad (15a)$$

$$D_{1-}X_{1-} + Q|A_{10}|^2(X_{1+} + X_{1-} + X_{2+} + X_{2-}) = 0, \quad (15b)$$

$$D_{2+}X_{2+} + Q|A_{20}|^2(X_{1+} + X_{1-} + X_{2+} + X_{2-}) = 0, \quad (15c)$$

$$D_{2-}X_{2-} + Q|A_{20}|^2(X_{1+} + X_{1-} + X_{2+} + X_{2-}) = 0, \quad (15d)$$

where the unknowns are $X_{1+} = \mathbf{A}_{10}^*\cdot\mathbf{A}_{1+}$, $X_{1-} = \mathbf{A}_{10}\cdot\mathbf{A}_{1-}^*$, $X_{2+} = \mathbf{A}_{20}^*\cdot\mathbf{A}_{2+}$, and $X_{2-} = \mathbf{A}_{20}\cdot\mathbf{A}_{2-}^*$. The coupling constant is

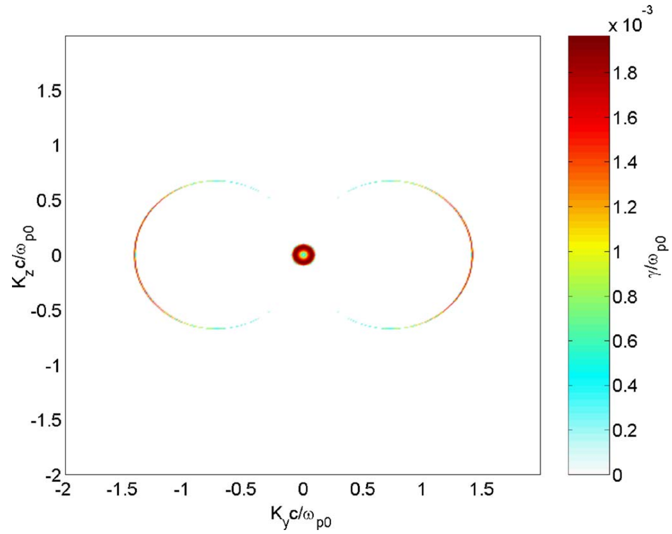

FIG. 1. (Color online) The growth rate γ/ω_{p0} as a function of the wave numbers K_y and K_z, for a single laser beam $\mathbf{A}_1$ propagating in the positive y direction with the wave number $(k_{1y}, k_{1z}) = (0.8, 0)\omega_{p0}/c$, having the amplitude $e|\mathbf{A}_{10}|/mc^2 = 0.1$. We used $n_{c0} = 0.7n_0$, $n_{h0} = 0.3n_0$, and $V_{Th} = 0.05c$.

$$Q = \frac{\omega_{p0}^2}{2}\left\{1 + \frac{K^2c^2}{\omega_{p0}^2}\frac{(k_{Dh}^2\Omega^2 - \omega_{pc}^2K^2)}{[(K^2 + k_{Dh}^2)\Omega^2 - \omega_{pc}^2K^2]}\right\}, \quad (16)$$

and the sidebands of the laser light are characterized by

$$D_{j\pm} = \pm 2[\omega_j\Omega - c^2\mathbf{k}_j\cdot\mathbf{K}] - K^2c^2, \quad (17)$$

where we have used that $\mathbf{v}_{gj} = c\mathbf{k}_j/\omega_j$.

Elimination of the unknowns from the system of equations (15) yields the nonlinear dispersion relation

$$\frac{1}{Q} + \left(\frac{1}{D_{1+}} + \frac{1}{D_{1-}}\right)|\mathbf{A}_{10}|^2 + \left(\frac{1}{D_{2+}} + \frac{1}{D_{2-}}\right)|\mathbf{A}_{20}|^2 = 0, \quad (18)$$

which relates the complex-valued frequency Ω to the wave number $\mathbf{K}$. Equation (18) covers the modulational instability as well as the wave backscattering instability against the EA perturbations. If either $|\mathbf{A}_{10}|$ or $|\mathbf{A}_{20}|$ is zero, then we recover the usual expressions for a single laser beam in a plasma.

III. NUMERICAL RESULTS

We have numerically solved the nonlinear dispersion relation (18) and presented our results in Figs. 1–3, where we have used the weakly relativistic pump wave amplitudes $e|A_{j0}|/mc^2 = 0.1$ with different sets of wave numbers for the two laser beams. For the scattering instabilities to be effective, we need to consider parameters for which the excited electron-acoustic wave is not strongly Landau damped. The criteria for linear electron-acoustic waves not to be strongly Landau-damped via wave-particle interactions have been found to be $T_c/T_h < 0.1$ and $n_{c0}/n_0 < 0.8$ [38,39]. In all cases, we used $n_{c0} = 0.7n_0$, $n_{h0} = 0.3n_0$, and $V_{Th} = 0.05c$, where $V_{Th} = (T_h/m)^{1/2}$ is the thermal speed. In Fig. 1, we have assumed a single beam A_1 (with $A_2 = 0$) that propagates in the y direc-

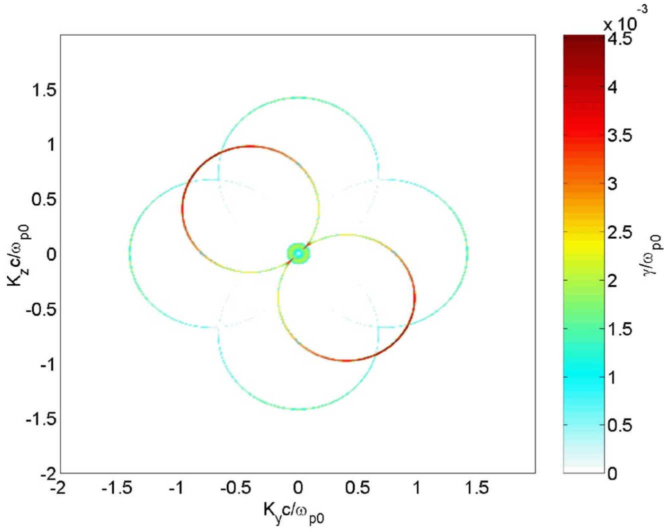

FIG. 2. (Color online) The growth rate γ/ω_{p0} as a function of the normalized wave numbers $K_y c/\omega_{p0}$ and $K_z c/\omega_{p0}$, for two coupled laser beams $\mathbf{A}_1$ and $\mathbf{A}_2$ propagating in the positive y and z directions, respectively, with wave numbers $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$ and $(k_{1y},k_{1z})=(0,0.8)\omega_{p0}/c$, respectively. Each beam has the amplitude $e|\mathbf{A}_{10}|/mc^2=e|\mathbf{A}_{20}|/mc^2=0.1$. We used $n_{c0}=0.7n_0$, $n_{h0}=0.3n_0$, and $V_{Th}=0.05c$.

tion, with the wave number $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$. For this case, we have a modulational and/or filamentation instability at small wave numbers, visible as a symmetric instability region near the origin in Fig. 1. We also have a parametric three-wave coupling for larger wave numbers, which essentially obeys the matching conditions $\omega_j=\omega_s+\Omega$ and $\mathbf{k}_j=\mathbf{k}_s+\mathbf{K}$, where ω_j and $\mathbf{k}_j$ are the frequency and the wave vector of the electromagnetic pump wave, ω_s and $\mathbf{k}_s$ are the frequency and the wave vector of the scattered and/or frequency downshifted electromagnetic daughter wave, Ω and $\mathbf{K}$ are the frequency and the wave vector of the EA perturbations, and where the light waves approximately obey the linear dispersion relations $\omega_j=(\omega_{p0}^2+k_j^2c^2)^{1/2}$ and $\omega_s=(\omega_{p0}^2+k_s^2c^2)^{1/2}$, while the EA perturbations follow the dispersion relation $\Omega=\omega_{pc}K/(K^2+k_{Dh}^2)^{1/2}$. We thus have the matching condition $(\omega_{p0}^2+k_j^2c^2)^{1/2}=[\omega_{p0}^2+(\mathbf{k}_j-\mathbf{K})^2c^2]^{1/2}+\omega_{pc}K/(K^2+k_{Dh}^2)^{1/2}$, which in two dimensions relates the components K_y and K_z of the electron-acoustic perturbations to each other, and which gives rise to narrow regions of instability for larger wave numbers, as seen in Fig. 1. This instability has a maximum growth rate $\gamma\approx 2\times 10^{-3}\omega_{p0}$ at $(K_y,K_z)\approx(1.3,0)\omega_{p0}/c$. For larger values of the pump wave number k_{1y}, the three-wave decay instability turns into a backscattering instability with a maximum growth rate at $K_y\approx 2k_{1y}$, and one also would have to take into account the Raman forward and backward scattering instabilities if the frequency of the laser beam is larger than twice the plasma frequency. The situation becomes more complex in the case of two coupled laser beams, as considered in Figs. 2 and 3. In Fig. 2, the two laser beams have the wave numbers $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$ and $(k_{2y},k_{2z})=(0,0.8)\omega_{p0}/c$, so that they propagate perpendicularly to each other. Here, we see new instabilities due to the parametric couplings between the two laser beams, which are visible as almost circular instability regions in Fig. 2. The largest growth rates now occur at $(K_y,K_z)\approx(0.8,-0.8)\omega_{p0}/c$ and $(K_y,K_z)\approx(-0.8,0.8)\omega_{p0}/c$, and we can also see maxima in the growth rate for small wave numbers at $(K_y,K_z)\approx(0.1,0.1)\omega_{p0}/c$. The case of two counter-propagating laser beams is illustrated in Fig. 3, where the two laser beams have the wave numbers $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$ and $(k_{2y},k_{2z})=(-0.8,0)\omega_{p0}/c$. Here, we see that the maximum growth rates are for wave vectors in the same directions as the beams, at $(K_y,K_z)\approx(1.6,0)\omega_{p0}/c$ and $(K_y,K_z)\approx(-1.6,0)\omega_{p0}/c$.

In order to investigate the dynamics of nonlinearly interacting laser beams in a two-electron plasma, we have performed numerical simulations of the system of Eqs. (12) and (13) in two spatial dimensions, and have displayed the results in Fig. 4. As an initial condition, we used that both A_1 and A_2 have constant amplitudes of $0.1mc^2/e$, and are propagating perpendicularly to each other. Due to symmetry reasons, it is sufficient to simulate one vector component of $\mathbf{A}_j$, which we denote A_j $(j=1,2)$. The background plasma density is slightly perturbed with a low-level noise (random numbers). In Fig. 4, we have considered two laser beams that are propagating perpendicularly to each other, corresponding to the case illustrated in Fig. 2. Here we see growing waves that are propagating obliquely to both laser beams, in correspondence to the instability analysis of Fig. 2; these waves have wavelengths of $\approx 5c/\omega_{p0}$ corresponding to a wave number of size ≈ 1.2, in agreement with the fastest growing waves in Fig. 2. During the exponential growth phase at $t=3750\omega_{p0}^{-1}$ (the middle row of panels in Fig. 4), the laser beams A_1 and A_2 are strongly correlated with each other, and the potential ϕ have maxima (corresponding to electron density minima) where the amplitudes of A_1 and A_2 have

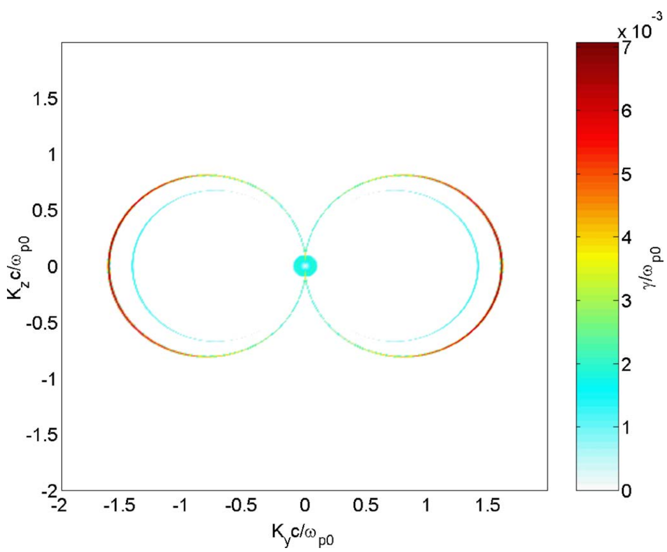

FIG. 3. (Color online) The growth rate γ/ω_{p0} as a function of the normalized wave numbers $K_y c/\omega_{p0}$ and $K_z c/\omega_{p0}$, for two counter-propagating laser beams $\mathbf{A}_1$ and $\mathbf{A}_2$ propagating in the positive and negative y directions, respectively, with wave numbers $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$ and $(k_{1y},k_{1z})=(-0.8,0)\omega_{p0}/c$, respectively. Each beam has the amplitude $e|\mathbf{A}_{10}|/mc^2=e|\mathbf{A}_{20}|/mc^2=0.1$. We used $n_{c0}=0.7n_0$, $n_{h0}=0.3n_0$, and $V_{Th}=0.05c$.

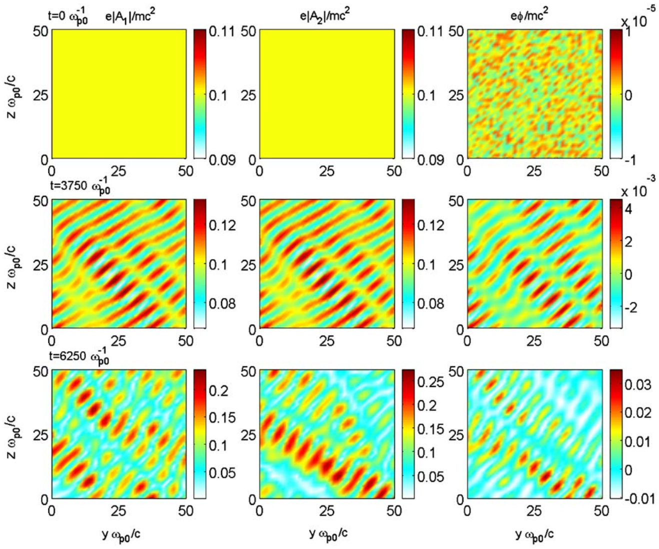

FIG. 4. (Color online) The dynamics of two coupled laser beams $\mathbf{A}_1$ and $\mathbf{A}_2$ that initially are propagating in the positive y and z directions, respectively, with wave numbers $(k_{1y},k_{1z})=(0.8,0)\omega_{p0}/c$ and $(k_{1y},k_{1z})=(0,0.8)\omega_{p0}/c$, respectively. Each beam has initially the amplitude $e|\mathbf{A}_{10}|/mc^2=e|\mathbf{A}_{20}|/mc^2=0.1$. (Same parameters as in Fig. 2.) The coupled beams decay into localized waves that are propagating obliquely towards larger y and z.

maxima. In the nonlinearly saturate phase, illustrated at time $t=6250\omega_{p0}^{-1}$ (the lower row of panels in Fig. 4), the two laser beams are no longer correlated, i.e., A_1 and A_2 do not have maxima at the same locations in space. The potential ϕ exhibit maxima where either A_1 or A_2 have their amplitude maxima. We note that the instability investigated here is different from the backscattering instability by a single laser beam. While the backscattering instability typically gives rise to electrostatic waves that are propagating in the same direction as the wave vector of the waves, the waves in Fig. 4 have almost zero phase speed along the wave vector. Instead, the groups of waves are propagating along the wave fronts from the lower left to the upper right parts of the panels in Fig. 4. Hence these waves will not trap particles in the same manner as for the backscattering instability considered by other authors [2,13] and new and interesting phenomena may occur on a kinetic level.

IV. SUMMARY

In summary, we have considered nonlinear interactions between two colliding intense laser beams and EA perturbations in a two-electron plasma containing high- and low-energy electrons. Such plasmas are frequently found in both laboratories and space, and show new features of nonlinear laser-plasma interactions in laboratory experiments. Our investigation of nonlinearly coupled two laser beams and EA perturbations reveals, to the best of our knowledge, that there are new classes of parametric instabilities, which are responsible for standing wave patterns that are absent for a single laser beam interacting with the plasma. It should be stressed that the two-fluid model for electron-acoustic perturbations in a collisionless plasma is valid for $V_{Tc}|\nabla| \ll \partial/\partial t \ll V_{Th}|\nabla|$, where V_{Tc} (V_{Th}) is the thermal speed of the cold (hot) electron component. This approximation remains intact if $n_{ec}/n_0 < 0.8$ and $T_c/T_h < 0.1$ [38,39]. Alternatively, one has to resort to a Vlasov treatment (including the radiation pressure) for the two-electron components participating in the electron-acoustic wave dynamics. However, this is beyond the scope of the present investigation.

The present results are thus useful in understanding the nonlinear propagation of two colliding electromagnetic beams in laser plasma experiments as well as in space plasmas on short time scales so that the ions do not have time to respond to laser beams and EA perturbations. Specifically, the nonlinear instabilities (both modulational and filamentation) of two coupled laser beams can produce light pipes that may cause local electron heating in a two-electron plasma. Hopefully, our theoretical and simulation results could be verified in forthcoming experiments devoted to laser-plasma interactions.

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[1] B. Bezzerides, D. W. Forslund, and E. Lindman, Phys. Fluids **21**, 2179 (1978).
[2] D. S. Montgomery, R. J. Focia, H. A. Rose, D. A. Russell, J. A. Cobble, J. C. Fernández, and R. P. Johnson, Phys. Rev. Lett. **87**, 155001 (2001).
[3] R. E. Ergun, Y. J. Su, L. Andersson, C. W. Carlson, J. P. McFadden, F. S. Mozer, D. L. Newman, M. V. Goldman, and R. J. Strangeway, Phys. Rev. Lett. **87**, 045003 (2001).
[4] M. Y. Yu and P. K. Shukla, J. Plasma Phys. **19**, 409 (1983).
[5] J. Soucek, V. Krasnoselskikh, T. Dudok de Wit, J. Pickett, and C. Kletzing, J. Geophys. Res. **110**, A08102 (2005).
[6] P. K. Shukla, L. Stenflo, and M. Hellberg, Phys. Rev. E **66**, 027403 (2002).
[7] P. K. Shukla and M. A. Hellberg, J. Plasma Phys. **67**, 363 (2002).
[8] P. K. Shukla, M. A. Hellberg, and L. Stenflo, J. Atmos. Sol.-Terr. Phys. **65**, 355 (2003).
[9] M. Y. Yu, K. H. Spatschek, and P. K. Shukla, Z. Naturforsch. A **29**, 1736 (1974).
[10] H. A. Rose and D. A. Russell, Phys. Plasmas **8**, 4784 (2001).
[11] P. K. Shukla and B. Eliasson, Phys. Rev. Lett. **94**, 065002 (2005).
[12] B. Eliasson and P. K. Shukla, Phys. Rep. **422**, 225 (2006).
[13] L. Yin, W. Daughton, B. J. Albright, B. Bezzerides, D. F.

DuBois, J. M. Kindel, and H. X. Vu, Phys. Rev. E **73**, 025401(R) (2006).
[14] N. L. Tsintsadze and L. Stenflo, Phys. Lett. **48A**, 399 (1974); P. K. Shukla and L. Stenflo, Phys. Rev. A **30**, 2110 (1984).
[15] P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **135**, 1 (1986).
[16] A. S. Sakharov and V. I. Kirsanov, Phys. Rev. E **49**, 3274 (1994).
[17] S. Guérin, G. Laval, P. Mora, J. C. Adam, A. Héron, and A. Bendib, Phys. Plasmas **2**, 2807 (1995).
[18] B. Quesnel, P. Mora, J. C. Adam, S. Guérin, A. Héron, and G. Laval, Phys. Rev. Lett. **78**, 2132 (1997).
[19] C. D. Decker, W. B. Mori, K. C. Tzeng, and T. Katsouleas, Phys. Plasmas **3**, 2047 (1996).
[20] H. C. Barr, P. Mason, and D. M. Parr, Phys. Rev. Lett. **83**, 1606 (1999).
[21] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
[22] M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972).
[23] R. Bingham, J. T. Mendonça, and P. K. Shukla, Plasma Phys. Controlled Fusion **46**, R1 (2004).
[24] C. J. McKinstrie and R. Bingham, Phys. Fluids B **4**, 2626 (1992).
[25] P. K. Shukla, Phys. Scr. **45**, 618 (1992).
[26] L. Bergé, Phys. Rev. E **58**, 6606 (1998).
[27] C. Ren, B. J. Duda, and W. B. Mori, Phys. Rev. E **64**, 067401 (2001).
[28] Q.-L. Dong, Z.-M. Sheng, and J. Zhang, Phys. Rev. E **66**, 027402 (2002).
[29] G. Shvets and N. J. Fisch, Phys. Rev. Lett. **86**, 3328 (2001).
[30] G. Shvets, Phys. Rev. Lett. **93**, 195004 (2004).
[31] K. Nagashima, J. Koga, and M. Kando, Phys. Rev. E **64**, 066403 (2001).
[32] Z.-M. Sheng, K. Mima, Y. Setoku, K. Nishihara, and J. Zhang, Phys. Plasmas **9**, 3147 (2002).
[33] J. J. Thomson, Nucl. Fusion **15**, 237 (1975).
[34] S. P. Obenschain, N. C. Luhmann, Jr., and P. T. Greiling, Phys. Rev. Lett. **36**, 1309 (1976).
[35] A. Y. Wong and R. G. Brandt, Radio Sci. **25**, 1251 (1990).
[36] P. Rodriguez *et al.*, Geophys. Res. Lett. **25**, 257 (1998).
[37] A. D. Ryutov, S. C. Cowley, and E. J. Valeo, Phys. Plasmas **8**, 36 (2001).
[38] P. Gary and R. L. Tokar, Phys. Fluids **28**, 2439 (1985).
[39] R. L. Mace and M. Hellberg, J. Plasma Phys. **43**, 239 (1990).